The More the Merrier: Efficient Multi-Source Graph Traversal

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ABSTRACT

Graph analytics on social networks, Web data, and communication networks has been widely used in a plethora of applications. Many graph analytics algorithms are based on breadth-first search (BFS) graph traversal, which is not only time-consuming for large datasets but also involves much redundant computation when executed multiple times from different start vertices. In this paper, we propose Multi-Source BFS (MS-BFS), an algorithm designed for running multiple concurrent BFSs over the same graph on a single CPU core that scales up as the number of cores increases. MS-BFS leverages the properties of *small-world networks*, which apply to many real-world graphs, and enables efficient graph traversal that: (i) shares common computation across concurrent BFSs; (ii) greatly reduces the number of random memory accesses during BFS; and (iii) does not incur synchronization costs. We demonstrate how the graph analytics application of computing the closeness centrality value of entities in a graph can be efficiently solved with MS-BFS. We also present an extensive experimental evaluation with both synthetic and real datasets, including Twitter and Wikipedia, showing that MS-BFS provides almost linear scalability with respect to the number of cores and excellent scalability for increasing graph sizes, outperforming state-of-the-art BFS algorithms by more than one order of magnitude when running a large number of BFSs.

1. INTRODUCTION

An ever-growing amount of information has been stored and manipulated as graphs. To better comprehend and assess the relationships between entities in this data, and to uncover patterns and new insights, graph analytics has become essential. Numerous applications have been extensively using graph analytics, including social network analysis, road network analysis, Web mining, and computational biology. A typical example in the field of social networks is identifying the most central entities, as these potentially have influence on others and, as a consequence, are of great importance to spread information, e.g., for marketing purposes [20].

In a wide range of graph analytics algorithms, including shortest path computation [14], graph centrality calculation [10, 27], and k-hop neighborhood detection [13], *breadth-first search* (BFS)-based *graph traversal* is an elementary building block used to systematically *traverse* a graph, i.e., to visit all the vertices and edges of the graph from a given start vertex. Because of the volume and nature of the data, BFS is a computationally expensive operation, leading to long processing times, in particular when executed in large datasets that are commonplace in the aforementioned fields.

To speed up BFS-based graph analytics, significant research has been done to develop efficient BFS algorithms that can take advantage of the parallelism provided by modern multi-core systems [2, 6, 8, 15, 19]. They optimize the execution of a single traversal, i.e., a single BFS, mostly by visiting and exploring vertices in a parallel fashion. Hence, previous work had to address not only parallelization-specific issues, such as thread synchronization and the presence of workload imbalance caused by skew, but also fundamental challenges in graph processing, including poor spatial and temporal locality in the memory access pattern [24]. Recent work on processing graphs in distributed environments—including scalable approaches [11, 12, 28, 30] as well as graph databases [33], and platforms for distributed analytics [16, 23, 25, 31]—can be used to span the execution of parallel graph traversals to multiple machines, improving the overall performance and coping with data that is partitioned across different nodes.

Although many graph analytics algorithms (e.g., shortest path computation on unweighted graphs) involve executing single BFSs and can make good use of the existing parallel implementations, there is a plethora of other applications that require hundreds (or even millions) of BFSs over the same graph—in many cases, one BFS is needed from each vertex of the graph; examples of such applications include calculating graph centralities, enumerating the neighborhoods for all vertices, and solving the all-pairs shortest distance problem. These scenarios do not fully benefit from current parallel BFS approaches. Often, the best one can do with existing approaches in order to reduce the overall runtime is to execute multiple single-threaded BFSs in parallel, instead of running parallel BFSs sequentially, because the former avoids synchronization and data transfer costs,

as we discuss in Section 6. Doing so, however, misses opportunities for sharing computation across multiple BFSs when the same vertex is visited by various traversals, which becomes inefficient for large graphs and hampers scalability.

In this paper, we propose *Multi-Source BFS* (MS-BFS), a new BFS algorithm for modern multi-core CPU architectures designed for graph analytics applications that run a large number of BFSs from multiple vertices of the same graph. MS-BFS takes an orthogonal approach from previous work: instead of parallelizing a single BFS, we focus on processing a large number of BFSs *concurrently in a single core*, while still allowing to scale up to multiple cores. This approach allows us to share the computation between different BFSs without paying the synchronization cost.

This work leverages properties of *small-world networks* [3]: the distance between any two vertices is very small compared to the size of the graph, and the number of vertices discovered in each iteration of the BFS algorithm grows rapidly. Concretely, this means that a BFS in such a network discovers most of the vertices in few iterations, and concurrent BFSs over the same graph have a high chance of having overlapping sets of discovered vertices in the same iteration. Based on these properties, in MS-BFS we *combine* accesses to the same vertex across multiple BFSs, which amortizes cache miss costs, improves cache locality, avoids redundant computation, and reduces the overall runtime. Note that small-world networks are commonplace as these properties apply to many real-world graphs, including social networks, gene networks, and Web connectivity graphs, which are of interest for many graph analytics applications.

MS-BFS executes concurrent BFSs in a data-parallel fashion that requires neither locks nor atomic operations, and it can be efficiently implemented using bit fields in wide CPU registers. We assume that the graph fits in main memory, which is a realistic assumption for many real-world graphs and applications (e.g., the Who to Follow service at Twitter [18]) as modern servers can store up to hundreds of billions of edges in memory. This design choice avoids overheads from disk accesses and network roundtrips, allowing unprecedented analytics performance. Nevertheless, the optimized variants of our algorithm as described in Sections 3.2 and 4.1.1 allows data to be scanned sequentially and thus can be easily adapted to provide good performance for disk-resident graphs as well.

MS-BFS is a generic BFS algorithm that can be applied to many graph problems that run multiple traversals from different start vertices; as an example of a real application, we demonstrate how it can be used to compute the closeness centrality value for all the graph vertices, a computationally expensive problem. We also present an extensive experimental evaluation using synthetic datasets generated with the LDBC Social Network Data Generator [9, 21], as well as real-world graphs from Twitter and Wikipedia, showing that MS-BFS (i) outperforms existing BFS algorithms by more than one order of magnitude when running multiple BFSs, and (ii) scales almost linearly with respect to graph size and number of cores. It is worth noting that the approach presented here was successfully used by the two leading teams—first and second place—in the 2014 SIGMOD Programming Contest.

Overall, our contributions are as follows:

- We propose Multi-Source BFS (MS-BFS), a graph traversal algorithm that can efficiently execute multiple concurrent BFSs over the same graph using a single CPU core (Section 3). MS-BFS is most efficient in small-world networks, where MS-BFS combines the execution of multiple BFSs in a synchronization-free manner, improving the memory access pattern and avoiding redundant computation. We also discuss additional tuning strategies to further improve the performance of the algorithm (Section 4).

- To demonstrate the feasibility of our approach, we show how a real graph analytics application—the computation of vertices' closeness centrality values—can be efficiently implemented using MS-BFS (Section 5).

- We present an extensive experimental evaluation with synthetic and real datasets, showing that MS-BFS scales almost linearly with an increasing number of CPU cores and that it exhibits excellent scalability with changes to the input graph size. We further show that MS-BFS greatly outperforms existing state-of-the-art BFS algorithms when running multiple BFSs (Section 6). We provide the full source code of all our implementations online.[1]

2. BACKGROUND

In this paper, we consider an unweighted graph $G = \{V, E\}$, where $V = \{v_1, \ldots, v_N\}$ is the set of vertices; $E = \{neighbors_{v_i}|_{i=1}^{N}\}$ is the set of edges; $neighbors_v$ is the set of vertices to which v connects (neighbor vertices of v); and N is the number of vertices in the graph, i.e., $N = |V|$. We further assume that G exhibits properties of *small-world networks* (Section 2.1), and that the graph analytics algorithms to be used over G use BFS-based graph traversal (Section 2.2).

2.1 Small-World Networks

In small-world networks, as the graph size increases, the average distance—the number of edges—between vertices increases logarithmically. In other words, we say that a graph G has *small-world properties* if its diameter, i.e., the longest distance between any two vertices in G, is low even for a large N [3]. Another property of many small-world graphs is that their distribution of *degree*, i.e., the number of neighbors of a vertex, follows the power law: concretely, few vertices have a very high number of neighbors, while most of the vertices have few connections. Graphs exhibiting the latter property are also known as *scale-free networks* [3].

A famous example of these properties is the six degrees of separation theory, suggesting that everyone is only six or fewer steps away from each other. Recent study shows that 92% of Facebook users ($N \approx 720$ million) are connected by only 5 steps, and that the average distance between users is 4.74 [5]. In fact, besides social networks, many other real-world graphs that are of critical interest for both academia and industry—including gene and neural networks, the world-wide web, wikis, movie-actor and scientific collaboration networks, as well as electrical power grids—exhibit small-world properties [3].

As we show in Section 3, we exploit small-world graph properties to provide an efficient implementation for our BFS approach.

[1]Source code available at `http://bit.ly/1B8u4uX`

2.2 Breadth-First Search Overview

Breadth-first search (BFS) traverses a graph G from a given start vertex, or *source*, $s \in V$. We present the original BFS algorithm, to which we refer as *textbook BFS*, in Listing 1. There are two main states for a vertex during a BFS traversal: *discovered*, which means that the BFS has already visited the vertex, and *explored*, which means that not only the vertex but also its edges and neighbors have been visited. The algorithm starts by adding s to *seen*, which is the set of vertices that have been discovered. It also adds the source vertex to *visit*, which is the set of vertices yet to be explored. By iterating over *visit* in Line 7, vertices in *visit* are explored to find new reachable vertices: vertices connected to v (Line 8) that have not been discovered yet (Line 9) are added to both *seen* and *visitNext*. Furthermore, newly discovered vertices are processed by the graph analytics application that uses the BFS (Line 12), e.g., a shortest path algorithm stores the distance between s and n. The *visitNext* set becomes the next *visit* set after all the vertices in the current one have been explored (Lines 13 and 14).

Note that, for every iteration in Line 6, *visit* only contains vertices that have the same distance, in number of edges, from the source s: we say that these vertices are in the same *BFS level*. The maximum number of levels that any BFS can have in G is equivalent to the diameter of G. Since G is a small-world network, its diameter is low, which means that a BFS will have a small number of levels as well: all vertices are discovered in few iterations, and the number of vertices discovered in each level grows rapidly. Table 1 shows this behavior in a synthetic dataset of 1 million vertices (generated with the data generator from LDBC), where a BFS is run over a connected component that comprises over 90% of the vertices of the graph. Note that the number of BFS levels is small compared to the size of the graph, and that nearly 95% of the vertices are discovered in two levels.

In a traditional implementation of the BFS algorithm, queue data structures are often used for *visit* and *visitNext*, while *seen* is represented by either a list or a hash set. The set E of edges is usually implemented as an adjacency list, where each vertex has its own list of neighbors, i.e., $neighbors_v$ is a list composed by all the neighbor vertices of v.

Optimizing BFS. Small-world graphs tend to have few connected components; often, the entire graph is a single component, which means that every vertex is reachable from every other vertex. As a consequence, the larger the traversed graph is, the more vertices and edges need to be visited by the BFS, which becomes a significantly time-consuming operation. This issue is exacerbated by its lack of memory locality, as shown in the random accesses to *seen* and to the adjacency list (Lines 8 and 9), reducing the usefulness of CPU caches. Furthermore, towards the end of the BFS execution, most of the vertices will have been already discovered (see Table 1), and there will be much fewer non-discovered vertices than vertices in the *visit* set; as a consequence, there will be a significant number of failed checks to *seen* (Line 9), which consumes resources unnecessarily [8]. Last, searching for vertices in *seen* and adding new ones become expensive if more efficient data structures are not used.

Optimizing the execution of the BFS algorithm for large datasets is essential for graph analytics, and there has been substantial work in this direction. Most of this work is focused on implementing a *single parallel BFS*, i.e., parallelizing a single BFS execution, mainly by making use of the *level-synchronous* approach: vertices are explored and discovered in parallel for each BFS level, i.e., Lines 7 and 8 are executed in parallel for each level. The main idea of this approach is to divide the work across multiple cores and thus speed up the execution of one BFS. However, this comes at a cost: *visit* and *visitNext* must be synchronized at the end of each BFS level (before a new iteration at Line 6 starts), and race conditions must be avoided when multiple threads are accessing *seen*.

Listing 1: Textbook BFS algorithm.

```
1  Input: G, s
2  seen ← {s}
3  visit ← {s}
4  visitNext ← ∅
5
6  while visit ≠ ∅
7      for each v ∈ visit do
8          for each n ∈ neighbors_v do
9              if n ∉ seen then
10                 seen ← seen ∪ {n}
11                 visitNext ← visitNext ∪ {n}
12                 do BFS computation on n
13     visit ← visitNext
14     visitNext ← ∅
```

For shared-memory and multi-core CPUs, numerous optimizations have been proposed to efficiently implement the level-synchronous approach and to address the foregoing challenges [2, 6, 7, 8, 15, 19], including the use of more efficient data structures, mechanisms to improve memory locality (e.g., sequential access to data structures [19]), and further optimizations specific to certain hardware architectures. Notably, Beamer et al. [8] propose a bottom-up approach to avoid many failed checks to *seen* as mentioned before. Instead of visiting new vertices by looking at the outgoing edges of discovered vertices, the approach iterates over non-discovered vertices, looking for an edge that connects it to a vertex that has already been discovered (i.e., that is in *visit*). The authors combine the textbook BFS with the bottom-up approach in a hybrid *direction-optimized* technique. Although their implementation is for single parallel BFSs, the optimization is mostly orthogonal to parallelization and can be used for sequential execution. We further use this technique to optimize our algorithm (Section 4.1.2) and for comparison purposes (Section 6).

Executing the BFS algorithm in distributed memory systems has also been extensively studied before [11, 12, 30], as this raises a new range of issues, including the need to

Table 1: Number of discovered vertices in each BFS level for a small-world network.

Level	N. Discovered Vertices	≈ Fraction (%)
0	1	< 0.01
1	90	< 0.01
2	12,516	1.40
3	371,638	41.20
4	492,876	54.60
5	25,825	2.90
6	42	< 0.01

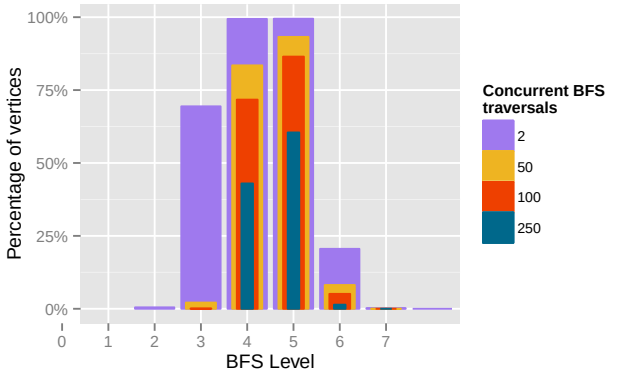

Figure 1: Percentage of vertex explorations that can be shared per level across 512 concurrent BFSs.

manage communication between CPUs and to partition the graph among processors, which is challenging and can deeply impact performance. Frameworks such as Parallel BGL [17], Pregel [25], Trinity [31], GraphLab [23], Giraph [4], and Teradata Aster [32] provide APIs to facilitate the scale-out of graph traversal and other graph algorithms to multiple nodes. Distributed graph databases, e.g., Titan [33], allow users to store and query graphs that are partitioned in multi-machine clusters, and engines such as Faunus [16] can be used on top of these databases to optimize the performance when doing large-scale graph analytics. Recently, there has also been an increasing interest in traversing and processing graphs using MapReduce [28].

3. MULTI-SOURCE BFS

As mentioned before, numerous graph analytics algorithms rely on executing multiple BFS traversals over the same graph from different sources. Often, a BFS traversal is run from every vertex in the graph. Clearly, this is very expensive, in particular for large real-world graphs that often have millions of vertices, and hence, require the execution of millions of BFSs. Our prime goal is then to optimize the execution of *multiple* BFSs on the same graph in order to improve the performance of such graph analytics applications. We focus on a non-distributed environment—a *single* server—and in-memory processing to exploit the capabilities of modern multi-core servers with large memories. This is reasonable even for graphs with hundreds of billions of edges, as shown by Gupta et al. [18], and provides a better performance per dollar for workloads that process multi-gigabyte datasets [29]. Note that these are not limitations of our algorithm, which can be extended to handle disk-resident graphs.

To the best of our knowledge, Multi-Source BFS (MS-BFS) is the first approach for efficiently executing a large number of BFSs over the same graph. Most of the existing approaches for multi-core CPUs, presented in the previous section, are orthogonal to our goal: they optimize the runtime execution of a *single* BFS, while we want to optimize the runtime execution for *multiple* BFSs. This brings a new range of requirements: (i) executing multiple traversals over the same graph exacerbates memory locality issues because the same vertices need to be discovered and explored for multiple BFSs, resulting in a higher number of cache misses; (ii) resource usage should be kept to a minimum to make the approach scalable for a large number of BFSs and as the number of cores increases; and (iii) synchronization costs of any kind should be avoided as their overheads become significantly apparent when executing vast amounts of BFSs.

Listing 2: The MS-BFS algorithm.

```
1  Input: G, 𝔹, S
2  seen_{s_i} ← {b_i} for all b_i ∈ 𝔹
3  visit ← ⋃_{b_i ∈ 𝔹} {(s_i, {b_i})}
4  visitNext ← ∅
5
6  while visit ≠ ∅
7      for each v in visit
8          𝔹'_v ← ∅
9          for each (v', 𝔹') ∈ visit where v' = v
10             𝔹'_v ← 𝔹'_v ∪ 𝔹'
11         for each n ∈ neighbors_v
12             𝔻 ← 𝔹'_v \ seen_n
13             if 𝔻 ≠ ∅
14                 visitNext ← visitNext ∪ {(n, 𝔻)}
15                 seen_n ← seen_n ∪ 𝔻
16                 do BFS computation on n
17     visit ← visitNext
18     visitNext ← ∅
```

To address these requirements, we (i) *share* computation across *concurrent* BFSs, exploiting the properties of small-world networks; (ii) execute hundreds of BFSs *concurrently* and using a *single core*, which scales up better than previous approaches; and (iii) use *neither locking nor atomic operations*, which makes the execution more efficient and also improves scalability.

In this section, we describe in detail our novel approach. We begin by introducing the algorithm in Section 3.1, and Section 3.2 shows how this algorithm can be mapped to efficient bit operations.

3.1 The MS-BFS Algorithm

An important observation about running multiple BFSs from different sources in the same graph is that every vertex is discovered multiple times—once for every BFS if we assume the graph has a single connected component—and every time the vertex is explored, its set of neighbors must be traversed, checking if each of them has already been discovered. This leads to many random memory accesses and potentially incurs a large number of cache misses.

To decrease the amortized processing time per vertex and to reduce the number of memory accesses, we propose an approach to *concurrently* run multiple BFSs and to *share* the exploration of vertices across these BFSs by leveraging the properties of small-world networks. Recall that the diameter of the graph is low—which means that the number of BFS levels is also small compared to the size of the graph—and that the number of discovered vertices in each level grows rapidly; since in few steps, all the vertices of the graph are discovered, we expect the likelihood of multiple concurrent BFSs having to explore the same vertices at the same level to be high. For a concrete example of this behavior, we analyze the LDBC graph with 1 million vertices introduced in the previous section. Figure 1 depicts, for every BFS level, the percentage of vertex explorations that can be shared by *at least* 2, 50, 100, and 250 BFSs out of 512 concurrent BFSs as indicated by the bar height and color. Note that the exploration of more than 70% of the vertices in levels 3 and 4 can be shared among *at least* 100 BFSs, and in level

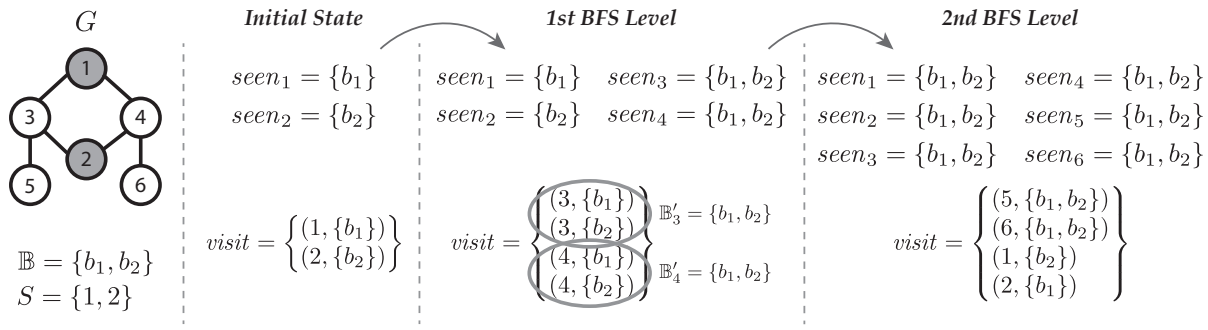

Figure 2: An example of the MS-BFS algorithm, where vertices 3 and 4 are explored once for two BFSs.

4, more than 60% of them can be shared by *at least* 250 BFSs. Concretely, this means that, in level 4, more than 60% of the vertices can be explored only once for 250 or more BFSs, instead of exploring them individually for each BFS. This can significantly reduce the number of memory accesses and speed up the overall processing.

We present the MS-BFS algorithm in Listing 2. In addition to the graph G, MS-BFS also receives as input the set of BFSs $\mathbb{B} = \{b_1, \ldots, b_\omega\}$, where each b_i represents a BFS, and ω is the total number of BFSs to be executed. Another input is the set $S = \{s_1, \ldots, s_\omega\}$ that contains the source vertex s_i for each BFS b_i. MS-BFS marks discovered vertices and vertices yet to be explored differently from the textbook BFS (Listing 1). Instead of having a single set *seen* of discovered vertices, each vertex v has its own set $seen_v \subseteq \mathbb{B}$ of BFSs that have already discovered it; furthermore, the sets *visit* and *visitNext* comprise tuples $(v', \mathbb{B}')$, where the set $\mathbb{B}' \subseteq \mathbb{B}$ contains the BFSs that must explore v'.

Similar to the textbook BFS, MS-BFS initially marks the source of each BFS as discovered (Line 2), and includes in *visit* each source with its corresponding BFS identifier to indicate which vertex needs to be explored for which BFS (Line 3). Next, for each BFS level (Line 6), the algorithm repeatedly selects a vertex v to be explored (Line 7) and merges all BFS sets from *visit* that refer to v into a set $\mathbb{B}'_v$ (Line 10). Thus, $\mathbb{B}'_v$ contains all BFSs that will explore v in the current level, and v can then be explored only *once* for all of them.

This shared exploration process is shown in Lines 11–16. For each neighbor n of v, the algorithm first identifies the set $\mathbb{D}$ of all the BFSs that need to explore n in the next level (Line 12). A BFS b_i must explore n if it belongs to the current level ($b_i \in \mathbb{B}'_v$) and if it has not discovered n yet ($b_i \notin seen_n$). If $\mathbb{D}$ is not empty, there are one or more BFSs that need to explore n in the next level. Hence, we add a tuple $(n, \mathbb{D})$ to *visitNext* (Line 14). Furthermore, we mark n as discovered (Line 15) and process it in the BFS computation (Line 15) for all BFSs in $\mathbb{D}$. Note that $neighbors_v$ is traversed only *once* for all $b_i \in \mathbb{D}$, and in the next level, each vertex n will be explored only *once* for these BFSs as well, which significantly reduces the number of memory accesses when running a large number of BFSs. Similar to the textbook BFS, *visitNext* is used as the *visit* set for the next BFS level (Lines 17 and 18).

Figure 2 shows an example of running MS-BFS for two BFSs, b_1 and b_2, starting from vertices 1 and 2 of graph G, respectively. At the beginning of the algorithm, all *seen* sets are initialized (we omit the empty ones), and *visit* contain the information that b_1 needs to explore vertex 1, and that b_2 needs to explore vertex 2. In the first BFS level, vertices 1 and 2 are explored, and since they are both adjacent to vertices 3 and 4, the *visit* set for the next level contains tuples for vertices 3 and 4 in BFSs b_1 and b_2. In the second BFS level, vertices 3 and 4 are explored. When picking vertex 3, Line 10 of the algorithm *merges* the sets of BFSs that need to explore 3 (see the highlighted section in Figure 2), resulting in $\mathbb{B}'_3 = \{b_1, b_2\}$. Therefore, 3 is explored only once for b_1 and b_2, and vertex 5 is discovered simultaneously by both BFSs as shown by the tuple $(5, \{b_1, b_2\})$ in *visit*. A similar process happens for vertex 4, as it is explored for both BFSs ($\mathbb{B}'_4 = \{b_1, b_2\}$), and for vertex 6, which is discovered simultaneously for both of them, adding the tuple $(6, \{b_1, b_2\})$ to the next *visit*. Note that during the second BFS level, vertex 1 is also discovered for b_2, and vertex 2 for b_1. Since all the vertices are already discovered at this point (i.e., all the *seen* sets contain the two BFSs), no tuples are added to *visit* in the third level, and the algorithm finishes.

Note that this approach differs from parallelizing a single BFS since MS-BFS still discovers and explores vertices sequentially. However, with MS-BFS, multiple BFSs are executed concurrently and share the exploration of vertices.

3.2 Leveraging Bit Operations

In practice, it is inefficient to run MS-BFS using set data structures as presented in Listing 2, since set operations are expensive when large numbers of concurrent BFSs are taken into account. In addition, *visit* needs to be entirely scanned every time a vertex v is picked in order to merge the sets of BFSs, which is prohibitively expensive for large graphs.

To solve these issues, we leverage *bit operations* to create a more efficient version of MS-BFS. In order to do so, we fix the maximum number of concurrent BFSs ω to a machine-specific parameter as elaborated in Section 4.2.1. This allows us to represent a set $\mathbb{B}' \subseteq \mathbb{B}$ of BFSs as a fixed-size bit field $f_1 \ldots f_\omega$ where $f_i = \texttt{1}$ if $b_i \in \mathbb{B}'$, and $f_i = \texttt{0}$ otherwise. Thus, we represent *seen* for a vertex v as $seen_v = f_1 \ldots f_\omega$, where $f_i = \texttt{1}$ if b_i has discovered v. Furthermore, *visit* for v is represented by $visit_v = f_1 \ldots f_\omega$, where $f_i = \texttt{1}$ if v needs to be explored by b_i; the same applies for *visitNext*.

The main advantage of representing BFS sets as bit fields is that MS-BFS can use efficient bit operations instead of complex set operations. Set unions $A \cup B$ become *binary or* operations $A \mid B$. Similarly, a set difference $A \setminus B$ becomes a *binary and* operation of A with the *negation* of B, i.e., $A \,\&\, {\sim}B$. We further denote an empty bit field as $\mathbb{B}_\varnothing$ and a bit field that only contains BFS b_i as $\texttt{1} << b_i$.

Listing 3: MS-BFS using bit operations.

```
1  Input: G, 𝔹, S
2  for each b_i ∈ 𝔹
3      seen[ s_i ] ← 1 << b_i
4      visit[ s_i ] ← 1 << b_i
5  reset visitNext
6
7  while visit ≠ ∅
8      for i = 1, ..., N
9          if visit[v_i] = 𝔹_∅, skip
10         for each n ∈ neighbors[v_i]
11             𝔻 ← visit[v_i] & ~seen[n]
12             if 𝔻 ≠ 𝔹_∅
13                 visitNext[n] ← visitNext[n] | 𝔻
14                 seen[n] ← seen[n] | 𝔻
15                 do BFS computation on n
16     visit ← visitNext
17     reset visitNext
```

To provide constant access to the bit fields of *visit*, *visitNext*, and *seen*, we store them in arrays sized to the number of vertices N in the graph. Thus, we have: $visit_v = visit[v]$, $visitNext_v = visitNext[v]$, and $seen_v = seen[v]$. In addition, we write $visit = \varnothing$ to represent the fact that $visit[v_i] = \mathbb{B}_\varnothing$ for all $i = 1, \ldots, N$.

Listing 3 presents the MS-BFS algorithm from Listing 2 using bit operations and array data structures. Note that the overall logic of MS-BFS does not change: Lines 7, 11, 13, and 14 from Listing 3 are equivalent to Lines 6, 12, 14, and 15 from Listing 2, respectively. A significant improvement from using bit fields to represent BFS sets and arrays for *visit* is that it avoids the expensive merging loop of Lines 9 and 10 from Listing 2.

We assume that the *neighbors* adjacency list is implemented as a single array that contains all the neighbors for all vertices, and that $neighbors[v_i]$ points to the memory block in *neighbors* that encompasses the neighbors of v_i. Also, these memory blocks are stored in order, i.e., $neighbors[v_{i-1}]$ precedes $neighbors[v_i]$ for all $i = 2, \ldots, N$. This representation improves memory locality for the algorithm: vertices are explored in order (Line 8), and as a consequence, the *neighbors* array can be retrieved in order as well (Line 10), which maximizes opportunities for sequential reads and makes a better use of caching [19].

Figure 3 shows the example presented in Figure 2 using arrays of bit fields for *visit* and *seen*. As in the previous figure, the *visitNext* array is similar to the *visit* array of the next BFS level and is then omitted for clarity. Each row represents the bit field for a vertex, and each column corresponds to one BFS. The symbol X indicates that the bit value is `1`; otherwise, the value is `0`.

Processing the initial *visit* array in the first BFS level, vertices 3 and 4 are discovered for both BFSs, since both of them are neighbors of sources 1 and 2. Hence, $seen[3]$ and $seen[4]$ have a bit field of value `11`, indicating that both BFSs have discovered the vertices. The bit fields in $visit[3]$ and $visit[4]$ have this value as well, indicating that these vertices need to be explored for both BFSs in the next level. During the second BFS level, vertices 3 and 4 are explored only once for both b_1 and b_2 (since $visit[3] = visit[4] = $ `11` at the end of the initial level), leading to the discovery of vertices 5 and 6 for both BFSs. As the *seen* array does not contain bits of value `0` anymore, no new vertices are discovered in the third BFS level.

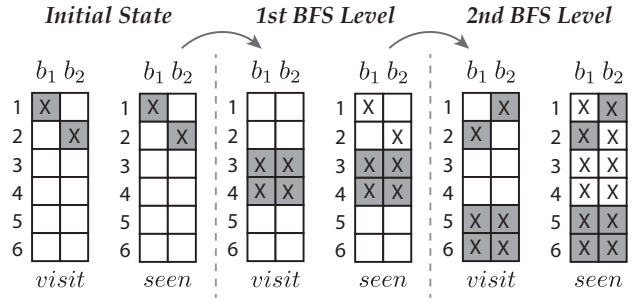

Figure 3: An example showing the steps of MS-BFS when using bit operations. Each row represents the bit field for a vertex, and each column corresponds to one BFS. In addition, the symbol X indicates that the value of the bit is `1`.

Note that our algorithm can leverage efficient native bit instructions, in particular when ω is a multiple of the machine's register width (see Section 4.2.1). This brings mainly two advantages: (i) our approach does not need explicit synchronization, as there is no competition for resources—the algorithm can run multiple concurrent BFSs in a *lock-free* and *atomic-free* fashion; and (ii) our approach is *nearly linear scalable* as more cores are added to execute a higher number of concurrent BFSs (Section 6.2 presents some results on this subject).

4. ALGORITHM TUNING

In this section, we discuss techniques to further improve the performance of MS-BFS, including techniques to improve memory locality and to avoid—even more—the impact of random memory accesses (Section 4.1), as well as efficient strategies to execute a number of BFSs greater than the size ω of the used bit fields (Section 4.2). In Section 6.2, we evaluate the performance impact of the presented optimizations.

4.1 Memory Access Tuning

4.1.1 Aggregated Neighbor Processing

Sequentially checking the elements in the *visit* array in Line 8 of Listing 3 improves the memory locality of MS-BFS and results in fewer caches misses, as mentioned earlier. However, there are still random accesses to the *visitNext* and *seen* arrays (Lines 11, 13, and 14) as the same neighbor vertex n can be discovered by different vertices and BFSs in the same level, i.e., $visitNext[n]$ and $seen[n]$ may be accessed in different iterations of Line 8. In addition, the application-specific BFS computation (Line 15) for n may have to be executed multiple times as well, which worsens the issue.

To provide further improvements in memory locality, we propose the *aggregated neighbor processing* (ANP) technique. The main idea is to reduce the number of both BFS computation calls and random memory accesses to *seen* by first collecting all the vertices that need to be explored in the next BFS level, and then by processing in batch the remainder of the algorithm, removing the dependency between *visit* and both *seen* and the BFS computation.

Listing 4 shows the MS-BFS algorithm using ANP. Concretely, when using ANP, we process a BFS level in two

Listing 4: MS-BFS algorithm using ANP.

```
1  Input: G, 𝔹, S
2  for each b_i ∈ 𝔹
3      seen[ s_i ] ← 1 << b_i
4      visit[ s_i ] ← 1 << b_i
5  reset visitNext
6
7  while visit ≠ ∅
8      for i = 1, ..., N
9          if visit[v_i] = 𝔹_∅, skip
10         for each n ∈ neighbors[v_i]
11             visitNext[n] ← visitNext[n] | visit[v_i]
12
13     for i = 1, ..., N
14         if visitNext[v_i] = 𝔹_∅, skip
15         visitNext[v_i] ← visitNext[v_i] & ∼seen[v_i]
16         seen[v_i] ← seen[v_i] | visitNext[v_i]
17         if visitNext[v_i] ≠ 𝔹_∅
18             do BFS computation on v_i
19     visit ← visitNext
20     reset visitNext
```

stages. In the first stage (Lines 8–11), we sequentially explore all vertices in *visit* to determine in which BFSs their neighbors should be visited, updating *visitNext*. In the second stage (Lines 13–18), we sequentially iterate over *visitNext*. Then, we update the bit fields of *visitNext* based on *seen*, and execute the BFS computation. Note that we only perform these steps *once* for every newly discovered vertex: we *aggregate* the neighbor processing.

ANP leverages the fact that *visitNext* is reset for every new BFS level. In addition, Lines 11 and 15 from Listing 4 are equivalent to Lines 11 and 13 from Listing 3 by means of the distributive property of binary operations; for a vertex n and vertices $v_1, \ldots, v_k$ of which n is neighbor:

$$\Big(visit[v_1] \mid \ldots \mid visit[v_k]\Big) \;\&\; \sim seen[n] \equiv \Big(visit[v_1] \;\&\; \sim seen[n]\Big) \mid \ldots \mid \Big(visit[v_k] \;\&\; \sim seen[n]\Big)$$

Note that random memory accesses to *visitNext* in Line 11 are inevitable and we discuss how to mitigate this issue in Section 4.1.3. Nevertheless, this technique brings a range of advantages. Notably, it: (i) reduces the number of memory accesses to *seen*, since the array is only retrieved once for every discovered vertex v, independent of the number of vertices of which v is a neighbor; (ii) replaces random access with sequential access to *seen*, which improves memory locality; and (iii) reduces the number of times that the BFS computation is executed. With these advantages, ANP improves the usage of low cache levels, prevents stalls caused by cache misses, and thus, reduces the overall execution MS-BFS time. As reported in Section 6.2, ANP speeds up MS-BFS by 60 to 110%.

4.1.2 Direction-Optimized Traversal

As we focus on small-world graphs, it is further beneficial to apply the *direction-optimized* BFS technique, introduced by Beamer et al. [8], to MS-BFS. The technique chooses at runtime between two BFS strategies: *top-down* and *bottom-up*. The former strategy is a conventional BFS, discovering new vertices by exploring the ones found in the previous level, i.e., by exploring the *visit* array in Lines 8–10 in Listing 3. In contrast, the bottom-up approach, when applied to MS-BFS, avoids traversing the *visit* array, and instead scans the *seen* array for vertices that have not been discovered by all BFSs yet. When such a vertex v is found, the approach traverses its edges and processes the *visit* entries of the neighbor vertices n that are adjacent to v:

$$visitNext[v] \leftarrow visitNext[v] \mid visit[n]$$

Note that, as suggested by the technique name, the two strategies work in different directions: the top-down one goes from discovered to non-discovered vertices, while the bottom-up one goes in the opposite direction. Direction-optimized traversal uses an heuristic based on the number of non-traversed edges during the BFS and a threshold to perform either the top-down or the bottom-up strategy. Concretely, the heuristic often chooses the top-down approach for the initial BFS levels, and the bottom-up approach for the final steps (where most of the vertices have already been discovered). The reader is referred to Beamer et al. [8] for further details.

Our experiments show that with both this hybrid approach and ANP, MS-BFS can significantly reduce the number of random accesses to *visit* and *visitNext*, improving the performance by up to 30%, see Section 6.2.

4.1.3 Neighbor Prefetching

Recall that the ANP technique reduces the number of random accesses to the *seen* array. However, many random accesses are still unavoidable when discovering neighbors and updating *visitNext* (Lines 10 and 11 in Listing 4).

To mitigate the high latency of these memory accesses, it is beneficial to use *prefetching*: once the vertex v_i is picked from *visit*, we detect its neighbors and the memory addresses of their entries in *visitNext*. We can then explicitly *prefetch* some of these entries *before* processing *visitNext* in the iteration of Line 10. As a consequence, this iteration is less prone to execution stalls because the prefetched *visitNext* entries are likely to be in the CPU cache when they are required, which provides an additional speedup to MS-BFS. Instead of doing the prefetching interleaved with the algorithm execution, it is also beneficial to do it asynchronously in simultaneous multithreading cores [22]. We identified experimentally that, by prefetching tens or even hundreds of neighbors, the performance of MS-BFS can be improved by up to 25%, as elaborated in Section 6.2.

4.2 Execution Strategies

4.2.1 How Many BFSs?

Conceptually, the MS-BFS algorithm can be used to run any number of concurrent BFSs by using bit fields of arbitrary sizes. However, our approach is more efficient when the bit operations are implemented using native machine instructions, which means that ω should be set according to the register and instruction width of the used CPU for optimal performance. As an example, on modern Intel CPUs, there are instructions and registers with a width of up to 256 bits, thus allowing MS-BFS to efficiently execute 256 concurrent BFSs; in CPUs supporting the AVX-512 extension, instructions exist that operate on 512 bits, which doubles the number of concurrent BFSs that can be executed using a single CPU register per vertex and data structure.

Table 2: Memory consumption of MS-BFS for N vertices, size ω of bit fields, and P parallel runs.

N	ω	P	Concurrent BFSs	Memory
1,000,000	64	1	64	22.8 MB
1,000,000	64	16	1,024	366.2 MB
1,000,000	64	64	3,840	1.4 GB
1,000,000	512	1	512	183.1 MB
1,000,000	512	16	8,192	2.9 GB
1,000,000	512	64	30,720	11.4 GB
50,000,000	64	64	3,840	71.5 GB
50,000,000	512	64	30,720	572.2 GB

Nevertheless, it is often the case that applications need to run BFSs for a number of sources greater than the size of any CPU-optimized ω. In this case, there are three different strategies that can be used: (1) increase ω by using multiple CPU registers for the bit fields, (2) execute multiple MS-BFS runs in parallel, and (3) execute multiple MS-BFS runs sequentially. In the first approach, multiple CPU registers are used to represent the bit fields in *seen*, *visit*, and *visitNext*, i.e., ω is set to a multiple of the register width. As an example, we can leverage two 128-bit registers to have 256-bit fields, which, in turn, enables us to run 256 BFSs concurrently. The main advantage of this approach is that, clearly, the graph needs to be traversed less often as more BFSs are executed simultaneously, thus allowing additional sharing of vertex explorations. Moreover, when these registers are stored adjacent in memory, they can be aligned to cache line boundaries so that accessing part of the bit field ensures that its other parts are in the CPU cache as well. Thus, we further reduce the number of main memory accesses during a MS-BFS run. In Section 6.2, we show that having bit fields that are exactly sized to fit a cache line exhibit the best performance. On current Intel CPUs, cache lines are 64 bytes wide, allowing efficient processing of 512 sources per MS-BFS run.

A second approach for executing a larger number of BFSs is to make use of parallelism. While the presented MS-BFS algorithm runs in a single core, multiple cores can be used to execute multiple MS-BFS runs in parallel since these runs are independent from each other. As a result, MS-BFS scales almost linearly with an increasing number of cores.

The drawback of the first two approaches is their potentially high memory consumption: for P parallel runs and N vertices, MS-BFS requires $P \times 3 \times N \times \omega$ bits of memory to store the fields for *seen*, *visit* and *visitNext*. Table 2 gives some examples of the total memory consumption for different graph sizes and numbers of parallel runs, running from hundreds to tens of thousands of BFSs concurrently.

Last, it is possible to execute multiple MS-BFS runs sequentially, since, again, runs are independent. This is particularly interesting when memory becomes an issue. Based on the available memory and to adapt to different situations, we can choose the best strategy and *combine* the three approaches. For instance, multiple threads, each using bit fields that are several registers wide, can be used to execute sequences of MS-BFS runs.

4.2.2 Heuristic for Maximum Sharing

When executing a number of BFSs greater than ω, it is useful to group BFSs in the same MS-BFS run (i.e., in the same set $\mathbb{B}$) that will share most computations at each level. Recall that the main idea of MS-BFS is to share vertex explorations across concurrent BFSs. As a consequence, the more BFSs explore the same vertex v in a given level, the less often v will have to be explored again in the same run, and the faster MS-BFS becomes.

The first clear approach to allow sharing of vertex explorations is to group BFSs based on their connected components: BFSs running in the same component should be in the same MS-BFS run, as different components do not share any vertices or edges. To optimize the sharing in a single connected component, we propose a heuristic to group BFSs based on ordering their corresponding source vertices *by degree*. Recall that small-world networks are often scale-free as well, which means that there are few vertices with a high degree, while most of the vertices have a significantly smaller number of neighbors. Based on this property and the fact that small-world networks have a low diameter, our intuition is that vertices with higher degrees will have a significant number of neighbors in common. Therefore, this heuristic comprises sorting the source vertices by descending order of degree, and then grouping BFSs according to this order to improve the sharing of vertex explorations. We expect that a MS-BFS run starting from the highest degree vertices will have a very efficient execution. In fact, our evaluation in Section 6.2 shows that, compared with a random assignment of BFSs to MS-BFS runs, this heuristic can improve the performance by up to 20%.

5. APPLICATION: COMPUTING CLOSENESS CENTRALITY

Since our approach executes multiple concurrent BFSs, MS-BFS must handle the application-specific computation (Line 15 in Listing 3) for multiple BFSs as well, which needs to be implemented efficiently. As many algorithms are based on determining the number of neighbors found in a BFS level, we elaborate how this is done in MS-BFS and describe a BFS computation to solve a real-world graph analytics problem—calculating the closeness centrality value for all the vertices in a graph. i.e., the *all-vertices closeness centrality* problem.

Closeness Centrality. Computing vertex centrality metrics is an important application in graph analytics. Centrality metrics can be used, for instance, to gain knowledge about how central persons are distributed in a social network, which can, in turn, be used for marketing purposes or research in the network structure. In the literature, many centrality metrics have been proposed, including closeness centrality [27] and betweenness centrality [10]; for this section, we focus on the former.

The closeness centrality value of a vertex v measures how close v is, in terms of shortest path, from all other vertices in the graph. Essentially, it is based on the inverse of the sum of the distances between v and all other vertices. From Wasserman and Faust [35]:

$$ClosenessCentrality_v = \frac{(C_v - 1)^2}{(N-1) * \Sigma_{u \in V} d(v,u)}$$

where C_v is the number of vertices in the connect component of v, and $d(v,u)$ denotes the geodesic distance (i.e., the length of the shortest path) between v and u. To compute $d(v,u)$ for all $u \in V$ in an unweighted graph, a BFS-based traversal from v is required to calculate and maintain

the geodesic distance in the BFS computation. For the *all-vertices closeness centrality* problem, a BFS must be run for every vertex in the graph, which makes this computationally expensive. We use this problem as an example of the applicability of MS-BFS in a real-world application.

Using MS-BFS. We can leverage MS-BFS to efficiently solve the all-vertices closeness centrality problem as it enables running hundreds of concurrent BFSs per core to calculate the required geodesic distances. Note that the algorithm does not need to store each distance $d(v, u)$ since it suffices to find the sum of all distances computed during the BFSs. An approach to find this sum is to count the number of discovered vertices for each BFS level, multiply this number by the current distance from the source, and then to finally sum all obtained multiplication results. Note that this needs to be done for each concurrent BFS.

Performing such counting of discovered vertices arises as an issue when using MS-BFS. For every discovered vertex, each BFS must determine whether this vertex belongs to it by detecting if the bit field *visitNext*$[v]$ contains a bit of value `1` for it. If so, a BFS-local counter must be incremented. This approach takes time $O(n * \omega)$, where n is the number of discovered vertices in the level.

An alternative is to use hardware operations that count the number of leading or trailing bits `0` in a bit field: we continuously find the position of a bit `1` using such an operation, increase the respective BFS-local counter, and set the bit to `0`, until there are no more bits `1` in the bit field. This approach results in $O(n * m)$ time, where $m \leq \omega$ is the number of BFSs that discovered the vertices in that level, i.e., the number of bits `1` in the *visitNext* bit fields. However, this is still inefficient when most of the bits in a field have value `1`, and also because this operation is sensitive to branch mispredictions, since the CPU cannot predict well whether there will be a new iteration over the bit field.

In order to provide a more efficient solution, we designed a $O(n)$ algorithm that efficiently updates BFS-local neighbor counters in a branch-free manner. Our general idea is to use a space-efficient 1 byte wide neighbor counter for every BFS. We update these counters every time the BFS computation is executed and copy their information to wider fields once they overflow. The main difference to the previous approaches is that we update 1-byte counters using SIMD instructions available in modern CPUs: by masking the *visitNext*$[v]$ bit field, we add 1 to every counter belonging to a BFS that discovered v in the current level and leave other counters unmodified. Note that, since the runtime of this BFS computation step is independent from the number of BFSs that discover v, it optimally leverages the benefits of ANP as presented in Section 4.1.1, which reduces the number of executions of the BFS computation. Due to the lack of space, we omit further details of our BFS-computation approach.

6. EXPERIMENTAL EVALUATION

To assess the efficiency of our approach and the presented optimizations, we study the performance of MS-BFS using both synthetic and real datasets. Notably, we report experiments on scalability with respect to input size, number of cores, and number of BFSs (source vertices), and discuss the impact of the tuning techniques introduced in Section 4.

Table 3: Properties of the evaluated datasets.

Graph	Vertices (k)	Edges (k)	Diameter	Memory
LDBC 50k	50	1,447	10	5.7 MB
LDBC 100k	100	5,252	6	20.4 MB
LDBC 250k	250	7,219	10	28.5 MB
LDBC 500k	500	14,419	11	56.9 MB
LDBC 1M	1,000	81,363	8	314 MB
LDBC 2M	2,000	57,659	13	228 MB
LDBC 5M	5,000	144,149	13	569 MB
LDBC 10M	10,000	288,260	15	1.14 GB
Wikipedia	4,314	112,643	17	446 MB
Twitter	41,652	2,405,026	19	9.3 GB

Also, we compare the performance of MS-BFS with the textbook BFS and a state-of-the-art BFS algorithm.

6.1 Experimental Setup

BFS Algorithms. In our experimental evaluation we use a number of different BFS implementations for comparison purposes: (i) MS-BFS for the CPU register widths 64, 128, and 256 bits; (ii) a non-parallel version of the Direction-Optimized BFS (DO-BFS) [8], a state-of-the-art BFS algorithm; and (iii) Textbook BFS (T-BFS) as shown in Listing 1.

For each MS-BFS variant we assess the performance when using a single register as well as using multiple registers for a single bit field to fill an entire cache line. We indicate the latter using the suffix *CL* and follow the approach explained in Section 4.2.1. We enable all other optimizations from Section 4 in our experiments with MS-BFS unless otherwise noted.

We performed our comparisons by using each of the algorithms to compute *all-vertices closeness centrality*, which, as described before, is a computationally expensive graph analytics problem that uses BFS-based graph traversal. It is worth mentioning that the overheads for computing the closeness centrality values are similar among the BFS algorithms in order to provide a fair comparison.

Other Competitors. Note that we do not compare MS-BFS with parallel BFS implementations as they are not optimized for the efficient execution of large number of BFSs. In fact, we implemented and experimented with the single-socket version of the parallel BFS introduced by Agarwal et al. [2], where the authors propose a parallel BFS implementation that uses a bitmap for the *seen* data structure, as well as efficient atomic operations to amortize the synchronization costs of the level-synchronous parallelization. When varying the number of cores, running single-threaded T-BFSs or DO-BFSs showed a significantly better BFS throughput than sequentially executing the same number of parallel BFSs to solve the all-vertices closeness centrality problem; due to a lack of space we cannot provide the complete results here. The main reason for this throughput difference is that, although highly optimized, the synchronization costs of parallel BFSs hinder good scalability for running a large number of BFSs.

As a further competitor we experimented with the well-known open-source graph database Neo4j [26]. We used their integrated closeness centrality computation function and benchmarked its hot-cache runtime. On the LDBC 50k graph with 50,000 vertices and 1.5 million edges the

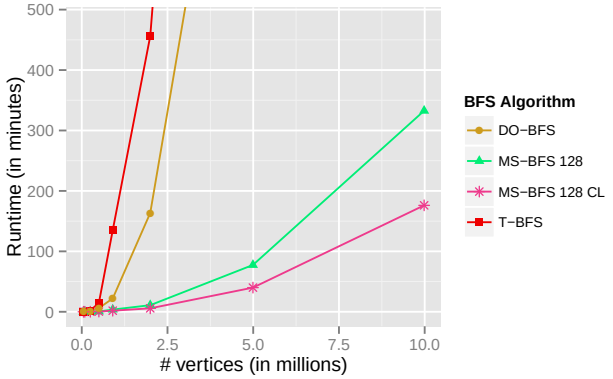

Figure 4: Data size scalability results.

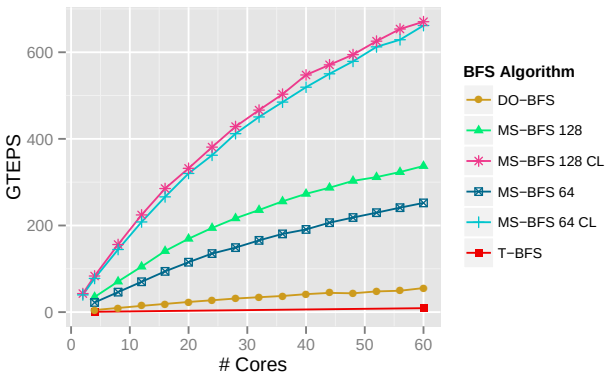

Figure 5: Multi-core scalability results.

all-vertices closeness centrality computation took 23 hours. Because Neo4j only used a single CPU core, and assuming perfect scalability we estimate the parallelized runtime on our evaluation machine would be 23 minutes which is still more than two orders of magnitude slower than the Textbook BFS we show. Compared to MS-BFS it is nearly four orders of magnitude slower. We, thus, do not include Neo4j in our comparison.

Software and Hardware Configuration. We ran the majority of our experiments on a 4-socket server machine with Intel Xeon E7-4870v2 CPUs, that has a total of 60 cores clocked at 2.30 GHz and with a Turbo Boost frequency of 2.90 GHz. The server is equipped with 1 TB of main memory equally divided over four NUMA regions. The experiments for Figures 6 and 7 were run on a machine equipped with an Intel Core i7-4770K CPU clocked at 3.50 GHz with a Turbo Boost frequency of 3.9 GHz; we use this CPU as it supports the AVX-2 instruction set for bit operations on 256-bit wide registers. All algorithms were implemented in C++ 11, compiled with GCC 4.8.2, and executed on Ubuntu 14.04 with kernel version 3.13.0-32.

Datasets. In our evaluation, we experimented with both synthetic and real datasets. For the former, we used the LDBC Social Network Data Generator [9, 21], which was designed to produce graphs with properties similar to real-world social networks. With this generator, we created synthetic graphs of various sizes—from 50,000 to 10 million vertices, and with up to 288 million edges. Additionally, we evaluated the performance of MS-BFS running on two real-world datasets from Twitter and Wikipedia. The Twitter dataset [34] contains 2.4 billion edges following the relationships of about 41 million users, while the Wikipedia dataset [36] represents a snapshot of the data as of July 2014 consisting of articles and links connecting them. Note that we consider the edges in all datasets as undirected. Table 3 shows the properties of the graphs used in our evaluation, including number of vertices and edges, diameter, and memory consumption of the used graph data structures.

6.2 Experimental Results

Data Size Scalability. To understand how MS-BFS scales as the size of the graph increases, we measured its runtime for different synthetic datasets, containing up to 10 million vertices and 288 million edges. Figure 4 shows the scalability of the BFS algorithms for all LDBC datasets we introduce before. The runtimes are measured in minutes and do not include loading times. The algorithms were executed using 60 cores, i.e., multiple runs of each algorithm were executed in parallel using all the cores available in the machine.

From the results, it is clear that T-BFS and DO-BFS do not scale well as the data size increases when running multiple BFSs. As an example, T-BFS and DO-BFS take 135 minutes and 22 minutes, respectively, to process the LDBC graph with 1 million vertices, while MS-BFS takes only 1.75 minutes. MS-BFS shows excellent scalability for all presented graphs and makes computations feasible on a single machine that are out of reach with traditional approaches: calculations that formerly took hours are sped up to minutes.

The results show the benefits of sharing computation among multiple concurrent BFSs. Even for large graphs (which means more BFS levels), MS-BFS has a good runtime as a significant amount of vertex explorations is shared. Also, our use of bit operations provides very efficient concurrency BFS execution in a single core. MS-BFS runs many concurrent BFSs, while T-BFS and DO-BFS can only perform one traversal per execution. In Figure 4, we can also show that using an entire cache line for bit fields in MS-BFS significantly improves the algorithm's performance. Our evaluation machine uses 512 bit wide cache lines, which we fill using the data from 4 128-bit registers fill, thus allowing the execution of 512 concurrent BFSs in a single core.

Multi-Core Scalability. In Figure 5, we show the scalability of MS-BFS, T-BFS and DO-BFS with increasing number of CPU cores. Instead of showing the execution runtime, we measure the performance in *traversed edges per second* (TEPS), i.e., the total number of edges considered for traversal divided by the runtime [1], when running all-vertices closeness centrality in LDBC 1M. We report the results in GTEPS, i.e., billion TEPS.

Up to 60 cores, Figure 5 depicts the nearly linear scalability of MS-BFS: by keeping the resource usage low for a large number of concurrent BFSs, the approach can execute more traversals as the number of cores increases. Notably, by using 128-bit registers and the entire cache line, MS-BFS can reach 670 GTEPS using 60 cores. T-BFS and DO-BFS shows a significantly lower performance.

MS-BFS does not exhibit an exact linear scale-up due to the activated Turbo Boost functionality in recent Intel CPUs. Turbo Boost increases the clock frequency of the CPUs when fewer cores are used, i.e., the more cores the algorithm uses, the lower the clock rate is. We chose not to disable this feature to show how the algorithm behaves

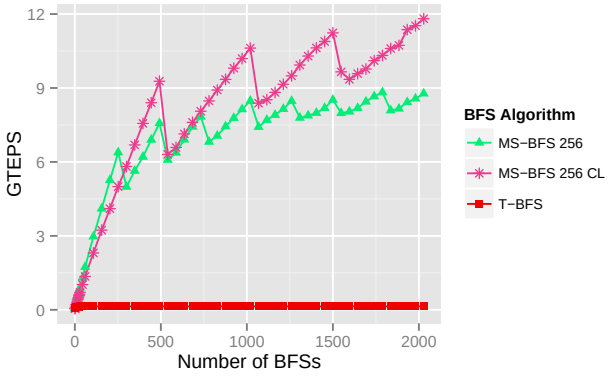

Figure 6: BFS count scalability results.

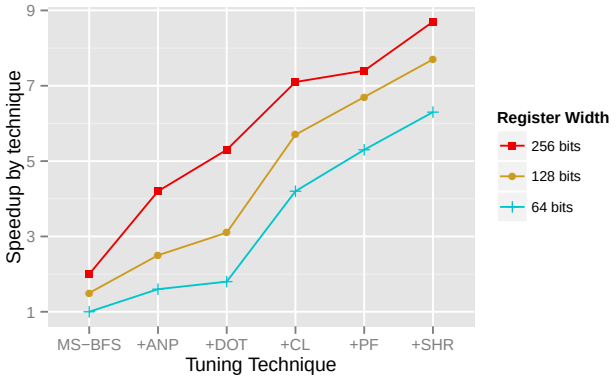

Figure 7: Speedup achieved by cumulatively applying different tuning techniques to MS-BFS.

under this functionality often used by server machines.

BFS Count Scalability. The main goal of MS-BFS is to execute a large number of BFSs efficiently. To evaluate this property, we study the scalability of MS-BFS as the number of BFSs increases. In Figure 6 we show the scalability from 1 to 2,000 closeness centrality computations. We, again, use the LDBC 1M dataset and report the results in GTEPS. In contrast to the previous experiment, this time we only use a single CPU core in order to make the results easier to understand. We note that we use same source vertices when comparing the different algorithms.

The number of traversed edges per second in T-BFS is constant, which is an expected result as BFSs are run sequentially. For MS-BFS, we see a different behavior: as more BFSs are executed, the GTEPS increase, since multiple BFSs can run concurrently in a single core. The peaks in the performance correspond to when the number of BFSs is a multiple of the bit field width of the MS-BFS run, which is 256 for MS-BFS 256, and 512 for MS-BFS 256 CL. The performance decays are related to the need for sequentially executing multiple MS-BFS runs as the bit fields become entirely filled. Nevertheless, by re-ordering the source vertices (Section 4.2.2), the performance keeps increasing as more BFSs are executed, which shows that MS-BFS provides good scalability with respect to the number of traversals.

Speedup. Table 4 shows the speedup of MS-BFS compared to T-BFS and DO-BFS when running all-vertices closeness centrality for two synthetic datasets as well as for the Wikipedia and Twitter datasets; in the Twitter dataset, we randomly selected 1 million vertices and compute the closeness centrality values for only these vertices. In these experiments, 60 cores were used. Some runs, indicated by an asterisk, were aborted after executing for more than eight hours, and the runtimes were then estimated by extrapolating the obtained results. From the results, we can see that MS-BFS outperforms both T-BFS and DO-BFS by factors of almost 2 orders of magnitude, varying from 12.1x to 88.5x.

Table 4: Runtime and speedup of MS-BFS compared to T-BFS and DO-BFS.

Graph	T-BFS	DO-BFS	MS-BFS	Speedup
LDBC 1M	2:15h	0:22h	0:02h	73.8x, 12.1x
LDBC 10M	*259:42h	*84:13h	2:56h	88.5x, 28.7x
Wikipedia	*32:48h	*12:50h	0:26h	75.4x, 29.5x
Twitter (1M)	*156:06h	*36:23h	2:52h	54.6x, 12.7x

*Execution aborted after 8 hours; numbers estimated.

Impact of Algorithm Tuning. To analyze the performance gains obtained by using each tuning technique described in Section 4, we evaluated their impacts by means of speedup. As the baseline, we use the MS-BFS algorithm as described in Section 3.2 using 64-bit registers. We then vary the register size and the techniques applied to the algorithm *cumulatively* and in the following order: aggregated neighbor processing (ANP), direction-optimized traversal (DOT), use of entire cache lines of 512 bits (CL), neighbor prefetching (PF) and heuristic for maximum sharing (SHR). The results are shown in Figure 7.

Using wider registers is beneficial for all optimizations, as more BFSs can be run concurrently. From the figure we can see that using entire cache lines (CL) technique provides the most significant speedup. ANP also shows a substantial speedup, in particular when using wide registers, which shows the impact of improving the memory locality for graph applications. Prefetching (PF) only shows noticeable speedup for smaller register sizes; it exhibits nearly no improvement when applied to MS-BFS using wide registers. Together, the tuning techniques improve the overall performance of MS-BFS by more than a factor of 8 over the baseline.

7. CONCLUSION

In this paper, we addressed the challenge of efficiently running a large number of graph traversals in graph analytics applications. By leveraging the properties of small-world networks, we proposed MS-BFS, an algorithm that can run multiple BFSs concurrently in a single core. MS-BFS reduces the number of random memory accesses, amortizes the high cost of cache misses, and takes advantage of wide registers and efficient bit operations in modern CPUs. We demonstrated how MS-BFS can be used to improve the performance of the all-vertices closeness centrality problem, and we are confident that the principles behind our algorithm can significantly help speedup a wide variety of other graph analytics algorithms as well. Our experiments reveal that MS-BFS outperforms state-of-the-art algorithms for running a large number of BFSs, and that our approach, combined with the proposed tuning techniques, provides excellent scalability with respect to data size, number of available CPUs, and number of BFSs.

There are numerous interesting directions for future work,

remarkably: redesign of MS-BFS for distributed environments and GPUs; new heuristics to maximize the computation sharing among BFSs; and the use of MS-BFS in query optimizers to batch BFS queries in order to improve the throughput of graph databases.

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